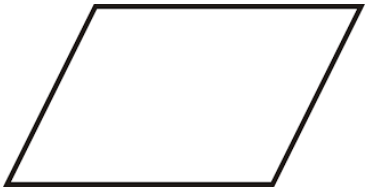
Geometry Reference Guide

Parallelogram

Definition: A parallelogram is a quadrilateral with two pairs of parallel sides.



Properties

- Opposite sides are congruent
- Opposite angles are congruent
- Diagonals bisect each other
- Consecutive angles are supplementary
- A square is a regular parallelogram

Rectangle

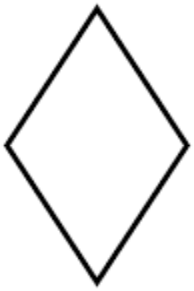
Definition: A rectangle is a parallelogram with four right angles.

Properties

- Diagonals are congruent
- All properties of a parallelogram apply

Rhombus

Definition: A rhombus is a quadrilateral with four congruent sides.



Properties

- Diagonals are perpendicular
- Diagonals bisect opposite angles
- All parallelogram properties apply
- A square is an equiangular rhombus

Kite

Definition: A kite is a quadrilateral with two pairs of congruent consecutive sides and no parallel sides.
Properties

- Non-vertex angles are congruent
- Diagonals are perpendicular
- The diagonal between vertex angles:
    - bisects those angles
    - bisects the other diagonal
    - creates two congruent triangles
    - is a line of symmetry

Trapezoid

Definition: A trapezoid is a quadrilateral with one pair of parallel sides.

Properties

- Base angles on the same leg are supplementary
- Midsegment: connects midpoints of the legs, parallel to the bases; its length is the average of the bases

Isosceles Trapezoid

- Legs are congruent
- Diagonals are congruent
- Base angles are congruent

Triangles

By Sides

- Scalene — no equal sides
- Isosceles — at least two equal sides
- Equilateral — three equal sides

By Angles

- Acute — all acute angles
- Right — one right angle
- Obtuse — one obtuse angle
- Equiangular — all 60°

Properties

- The longest side is opposite the largest angle
- The sum of any two sides is greater than the third
- *Exterior-Angle Theorem:* exterior angle = sum of the remote interior angles

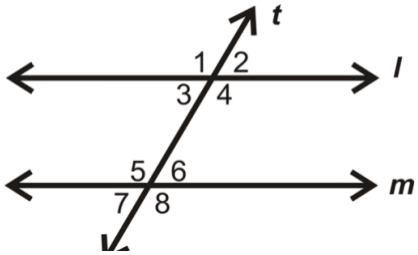
Special Segments

- Midsegment → joins side midpoints, parallel to third side, ½ its length
- Median → vertex to midpoint of opposite side
- Altitude → vertex perpendicular to opposite side

Points of Concurrency

Point	Description	Notes
Orthocenter	Intersection of altitudes	—
Circumcenter	Intersection of perpendicular bisectors	Equidistant from vertices
Incenter	Intersection of angle bisectors	Equidistant from sides
Centroid	Intersection of medians	Divides each median 2 : 1; center of balance

Angles with Parallel Lines



Angle Pair	Examples	Relationship
Corresponding	(1, 5), (2, 6), (3, 7), (4, 8)	Congruent
Alternate Interior	(3, 6), (4, 5)	Congruent
Alternate Exterior	(1, 8), (2, 7)	Congruent
Same-Side Interior	(3, 5), (4, 6)	Supplementary
Same-Side Exterior	(1, 7), (2, 8)	Supplementary

Vertical Angles: congruent
Linear Pair: supplementary

Circles

- Circumference: $C = 2\pi r = \pi d$
- Arc Length: $(a/360)\,2\pi r$ or θr
- Sector Area: $(a/360)\,\pi r^2$ or $\frac{1}{2}\theta r^2$
- Central Angle = Arc measure
- Inscribed Angle = ½ Arc measure

Polygons

Sum of Interior Angles:
$(n-2)\times 180°$

Each Interior Angle (regular polygon):
$(n-2)\times 180°/n = 180° - 360°/n$

Sum of Exterior Angles: 360°

Each Exterior Angle: $360°/n$

Area Formulas

Figure	Formula
Triangle	$A = \frac{1}{2} bh$ or $A = \sqrt{[s(s-a)(s-b)(s-c)]}$ or $A = ab \sin C$
Parallelogram	$A = bh$
Rectangle	$A = bh$
Square	$A = s^2$
Rhombus	$A = bh$ or $A = \frac{1}{2} d_1 d_2$
Kite	$A = \frac{1}{2} d_1 d_2$
Trapezoid	$A = \frac{1}{2} (b_1 + b_2) h$
Regular Polygon	$A = \frac{1}{2} a p$
Circle	$A = \pi r^2$

Surface Area

Solid	Formula
Prism	$SA = 2B + pH$
Pyramid	$SA = B + \frac{1}{2} p l$
Cylinder	$SA = 2\pi r^2 + 2\pi r H$
Cone	$SA = \pi r^2 + \pi r l$
Sphere	$SA = 4\pi r^2$
Hemisphere	$SA = 3\pi r^2$

Volume

Solid	Formula
Prism	$V = B\,H$
Cylinder	$V = \pi r^2 H$
Pyramid	$V = \frac{1}{3} B\,H$
Cone	$V = \frac{1}{3} \pi r^2 H$
Sphere	$V = 4/3\ \pi r^3$
Hemisphere	$V = 2/3\ \pi r^3$